

## 9<sup>th</sup> - Physics Gravitation



It has been observed that an object dropped from a height falls towards the Earth i.e. earth pulls everything towards itself. Newton generalized this idea and said that not only earth but every object in the universe attracts every other body. This force of attraction between two objects is called the force of gravitation or gravitational force.

**Gravitation:** defined as the non contact force of attraction between any two bodies in the universe. The earth attracts (or pulls) all objects lying on or near its surface towards its centre. The force with which the earth pulls the objects towards its centre is called gravitational force of the Earth or gravity of the Earth.

**The Universal Law of Gravitation:** formulated by Newton. The gravitational force of attraction between any two particles in the universe is directly proportional to the product of the masses of the particles and is inversely proportional to the square of the distance between the particles. The direction of the force is along the line joining the two particles. Let A and B be two particles of masses  $m_1$  and  $m_2$  respectively. Let the distance  $AB = r$ . By the law of gravitation, the particle A attracts the particle B with a force  $F$ , such that  $F \propto m_1 m_2$  (for a given separation between the particles)  $F \propto 1/r^2$  (for a given pair of particles). So,  $F \propto m_1 m_2 / r^2$  or  $F = G m_1 m_2 / r^2$

Here "G" is a constant known as the universal constant of gravitation. The value of G was experimentally measured in the laboratory by Cavendish, long after Newton's death. This value is  $G = 6.67 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$ . Its value does not depend on the medium between the two bodies and the masses of the bodies or the distance between them. This law is universal in the sense that it is applicable to all bodies, whether the bodies are big or small, whether they are celestial or terrestrial. Suppose the masses of two bodies having 1 kg each and the distance  $d$  between them is 1 m, then  $F = G [m_1 = m_2 = 1 \text{ kg and } r = 1 \text{ m}]$

Then, the universal gravitational constant is defined as the gravitational force between two objects of unit mass each separated by a unit distance from each other placed anywhere in space. The SI unit of G is  $\text{N m}^2 / \text{kg}^2$ . The value of G was found out by Henry Cavendish (1731-1810) by using a sensitive balance.

**Importance of Universal Law of Gravitation:** The universal law of gravitation successfully explained several phenomena given as below:

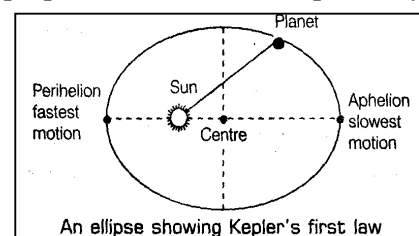
1. The force that binds us to the earth.
2. The motion of the moon around the earth.
3. The motion of planets around the sun.
4. The tides due to the moon and the sun.
5. The flow of water in rivers is also due to gravitational force of the earth on water.

**Motion of Moon around Earth and Centripetal Force:** The motion of the moon around the earth is due to centripetal force provided by the force of attraction of the earth. The force that keeps a body moving along the circular path is acting towards the centre and responsible for the change in direction of velocity or acceleration is called centripetal (centre seeking) force.

The motion of the moon around the earth is due to the centripetal force. The centripetal force is provided by the force of attraction of the earth. If there were no such force, then the moon would pursue a uniform straight line motion. If an object of mass  $m$  moves with a uniform speed ( $v$ ) along a circular path of radius ( $r$ ), its acceleration is  $a = v^2 / r$ . Then by Newton's second law of motion, the centripetal force is  $F = m v^2 / r$ .

**Kepler's laws of planetary motion:** Johannes Kepler proposed three laws of planetary motion in 16th century. The three laws are given as below:

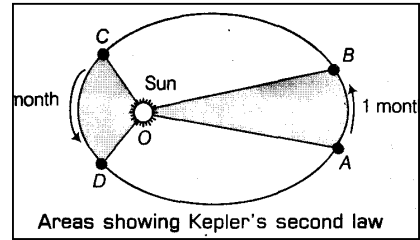
1. **Kepler's First Law:** it states that the path of any planet in an orbit around the sun follows the shape of an ellipse with the sun at one of its foci. The point in the





orbit of a planet nearest to the sun is called perihelion and the point farthest from the sun is called aphelion.

**2. Kepler's Second Law:** It states that an imaginary line from the sun to the planet sweeps out equal areas in equal intervals of time. Thus, if the time of travelling of a planet from A to B and from C to D is same, then the areas AOB and COD are equal.



**3. Kepler's Third Law:** It states that the cube of the mean distance of a planet from the sun is directly proportional to the square of its orbital period T. It is expressed as  $r^3 \propto T^2$ ,  $r^3 = K T^2$  or  $K = r^3 / T^2$ , where T = time period of the planet (around the sun), r = radius as mean distance of the planet from the sun and K = Kepler constant. Kepler could not give a theory to explain the motion of planets. Newton showed that the cause of the planetary motion is the gravitational force that the sun exerts on them. Newton used Kepler's third law to calculate the gravitational force of attraction. Suppose the orbital velocity is v and the radius of the orbit is r. Then, the force acting on an orbiting planet is given by  $F = mv^2 / r$ , where, m is the mass of planet.

If 'T' denotes the time period, then  $T = 2\pi r / v$  OR  $v = 2\pi r / T$

Substituting the value of v in Equation 1, we get

But according to Kepler's third law of planetary motion,  $r^3 = K T^2$  Or  $T^2 = r^3 / K$

$$F = \frac{m}{r} \left( \frac{2\pi r}{T} \right)^2 = 4\pi^2 m \frac{r}{T^2}$$

Putting this value into Equations, we get

$$F = \frac{4\pi^2 m \cdot r}{\left( \frac{r^3}{K} \right)} = \frac{4\pi^2 K m}{r^2}$$

**Free fall:** Whenever objects fall towards the earth under the earth's gravitational force alone, then these are called freely falling objects and such a motion is called free fall.

**Acceleration due to Gravity (g):** Whenever an object falls towards the earth, acceleration is involved. This acceleration is due to the earth's gravitational pull and is called acceleration due to gravity (or acceleration due to the gravitational force of the earth). It is denoted by g. The SI unit of 'g' is the same as that of acceleration, i.e. m/s<sup>2</sup>. Let mass of the earth be 'M' and an object falling freely towards it be 'm'. The distance between centres of the earth and the object is 'R.' From Newton's law of gravitation,  $F = GMm / R^2$

Also, from second law of motion, force exerted on object,  $F = ma$ . Since,  $a = g$  (i.e. acceleration due to gravity);  $F = mg$ . Equating RHS of Eqs. (i) and (ii), we get

$$mg = \frac{GMm}{R^2} \quad \text{or} \quad g = \frac{GM}{R^2}$$

From the formula, it is clear that acceleration due to gravity does not depend on the mass of a falling object. It depends only on the mass of the earth or celestial bodies. As distance of an object from the centre of the celestial body increases, the value of g decreases. Earth is flattened at poles. Thus, radius of the earth is less at poles than at equator. Hence, the value of g is less at equator than at poles.

Calculation of value of g: to calculate the value of g, we should put the values of G, M and R in the above formula i.e.  $g = GM / R^2$ . Mass of Earth  $M = 6 \times 10^{24}$  kg.

Radius of Earth  $R = 6.4 \times 10^6$  m. Universal gravitational constant,  $G = 6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$

$$g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} = 9.8 \text{ m/s}^2$$





**Motion of Objects under the Influence of Gravitational Force of the Earth:** the three equations of motion we have derived earlier is for bodies under uniform acceleration. Since, in case of motion of bodies under free fall, there is also a uniform acceleration, i.e. acceleration due to gravity ( $g$ ) acting downwards. So the previous three equations of motion can be applied for the motion of bodies under free fall as follows:

| General equations of motion    | Equations for body under free fall               |
|--------------------------------|--|
| 1. $v = u + at$                | $v = u + gt$ [ $\because a = g$ ]                |
| 2. $s = ut + \frac{1}{2} at^2$ | $h = ut + \frac{1}{2} gt^2$ [ $\because a = g$ ] |
| 3. $v^2 = u^2 + 2as$           | $v^2 = u^2 + 2gh$ [ $\because a = g$ ]           |

Where, 'h' is the height from which the object falls, 't' is the time fall, 'u' is the initial velocity and 'v' is the final velocity when body accelerates at  $g$ .

In solving numerical problems, we should remember the following points:

1. If an object falls vertically downwards, then acceleration due to gravity is taken as positive, since its velocity increases while falling.
2. If an object is thrown vertically upwards, then acceleration due to gravity is taken as negative, since its velocity decreases as it moves upward.

**Mass:** The mass is the total content of the body which measures the inertia of a body. It is a scalar quantity and its SI unit is kilogram. Irrespective of the position of the body in the universe, mass always remains constant everywhere. The mass of the body cannot be zero.

**Weight:** of an object is the force with which it is attracted towards the earth. Weight of an object,  $w = mg$ , where,  $m$  = mass,  $g$  = acceleration due to gravity.

$$w = \frac{GMm}{R^2}$$

Here,  $M$  = mass of the earth and  $R$  radius of the earth

From the above formula, it is clear that weight of an object will change on a planet other than the earth. Spring balance is used to measure the weight of a body and pan balance is used to measure the mass of a body.

**Important points regarding weight are as follow:**

1. Weight is a vector quantity; it acts in vertically downward direction, and its SI unit is Newton (N). Weight of 1 kg mass is 9.8 N (i.e. 1 kg-wt=9.8N).
2. Weight of an object is not constant; it changes from place to place.
3. In the space, where  $g = 0$ , weight of an object is zero.
4. At the centre of the earth, weight becomes zero. This is due to the fact that on going down to the earth value of  $g$  decreases and at the centre of the earth,  $g = 0$ .

**Weight of an Object on the Moon:** Let the mass of an object be 'm' and its weight on the moon be 'W<sub>m</sub>'. Suppose the mass of the moon is 'M' and its radius be 'R'. According to universal law of gravitation, the weight of an object on the moon will be

$$w_m = G \frac{M \times m}{R^2}$$

Let the weight of the same object on the earth be 'W<sub>e</sub>'. But the mass of the earth is 100 times that of the moon and the radius of the earth is 4 times that of the moon. Weight of the object on the earth,

On dividing these equations,

$$w_e = G \frac{100 M \times m}{(4R)^2} = G \frac{100M \times m}{16R^2}$$

$$\frac{w_m}{w_e} = G \frac{M \times m \times 16R^2}{R^2 \times G \times 100M \times m} = \frac{16}{100} \approx \frac{1}{6}$$

$$w_m = \frac{1}{6} w_e$$



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Thus, the weight of an object on the moon is one-sixth of its weight on the earth.

### Exercise

1. A man weighs 600 N on the earth. What is his mass, if  $g$  is  $10 \text{ m/s}^2$ ? On the moon, his weight would be 100 N. What is the acceleration due to gravity on the moon?
2. A particle weighs 120 N on the surface of the earth. At what height above the earth's surface will its weight be 30 N? Radius of the earth = 6400 km.
3. A particle is released from rest from a height. Find the distance it falls through in (a) 1 s, (b) 2 s, and (c) 3 s.
4. A person takes out a coin from his pocket and drops it from a height of 1.6 m. With what speed does it strike the ground?
5. A body is thrown upwards with a speed of 29.4 m/s. Find the height the particle rises through in (a) 1 s, and (b) 2 s.
6. Find the value of the acceleration due to gravity at a height of 12,800 km from the surface of the earth. Earth's radius = 6,400 km.
7. An object is dropped from height. It moves through a distance of 24.5 m in the last second before hitting ground. Find the height from which it was dropped. - [Ans. 44.1 m]
8. Mitali weighs 750 N on the earth.
  - a. On the planet mars, the force of gravity is 38% of that of the earth. How much will Mitali weigh on the mars?
  - b. What will be Mitali's mass on the earth? [Ans. (i) 285 N, (ii) 76.5kg]:
9. Weight of a girl on the earth's surface is 66 N. Find out the weight at the moon. [Ans. 11N]

