

# 10<sup>th</sup> – Polynomials I



A term 'monomial' is a constant or the product of a constant and one or more variables raised to a whole number exponent. For e.g: 15,  $3xy^2$ ,  $ac^3$ . A 'polynomial' is any finite sum of terms. For e.g.:  $7x^2 - 5x + 2$ ,  $2x^2y - 3xy^2$  are polynomials.

## Classification of Polynomial

1. Monomial: One term  $x$
2. Binomial: Two terms  $x + y$
3. Trinomial: Three terms  $x + y + xy$

Degree of Polynomial with one variable is the highest power to which the variable is raised in any one term.

Q. Write the degree of the following polynomial:

1.  $6x^3 - 5x^2 + 2x - 10$  (3)
2.  $5x - 3$  (1)
3.  $-30$  (zero)

## Value of a Polynomial at a Point

If we denote a polynomial by  $p(x)$  then by  $p(a)$  we mean the numerical value of the expression when  $a$  is substituted for  $x$ .

Q Find the value of  $p(x)$  at  $x = 2$  and  $x = 3$ :  $p(x) = x^3 - 5x^2 + 7x - 3$  (-1, 0)

## Zeroes of Polynomial

If  $p(x)$  is a polynomial and 'a' is a constant such that  $p(a) = 0$ , then 'a' is said to be a zero of polynomial  $p(x)$ .

## Location of Zeroes of Polynomial

If we plot the graph of  $y = p(x)$  on a graph paper then 'a' is a zero of  $p(x)$  if and only if  $(a, 0)$  lies on the graph of  $y = p(x)$ .

## Relationship between zeroes and coefficients of the polynomial

If  $p(x) = ax + b$ ,  $a \neq 0$  and ' $\alpha$ ' (alpha) is a zero of  $p(x)$ , then:

$$\alpha = -b/a = -\text{constant term/ coefficient of } x$$

For a quadratic polynomial  $p(x) = ax^2 + bx + c$ ,  $a \neq 0$  and if  $\alpha$  (alpha) and  $\beta$  (beta) are the two zeroes of the polynomial:

$$\text{Then sum of zeroes: } \alpha + \beta = -b/a = -\text{coefficient of } x / \text{Coefficient of } x^2$$

$$\text{The product of zeroes: } \alpha \beta = c/a = \text{constant term/ coefficient of } x^2$$

We can use these relationships to obtain values of several expressions involving  $\alpha$  (alpha) and  $\beta$  (beta) in terms of  $a$ ,  $b$ ,  $c$ .

## Practice Session I

Q.1 Find the zeroes of the following quadratic polynomial and verify the relationship between the zeroes and the coefficients.

- (A)  $x^2 - 5x + 6$  (B)  $4x^2 + 24x + 35$  (C)  $4x^2 - 1$  (D)  $3x^2 + 5x - 2$   
(E)  $x^2 + 5x + 4$

$$(2, 3), (-5/2, -7/2), (-1/2, 1/2), (-2, 1/3), (-1, -4)$$

Q2. Find a quadratic polynomial each with the given numbers as the sum and product of zeroes respectively.

- (A)  $4/3, 1/3$  (B)  $16/7, 4/7$  (C)  $-2/15, -1/15$  (D)  $13, 22$  (E)  $0, -16$

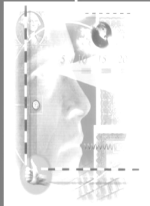
$$(3x^2 - 4x + 1), (7x^2 - 16x + 4), (15x^2 + 2x - 1), (x^2 - 13x + 22), (x^2 - 16)$$

## Plotting a Polynomial of the graph:



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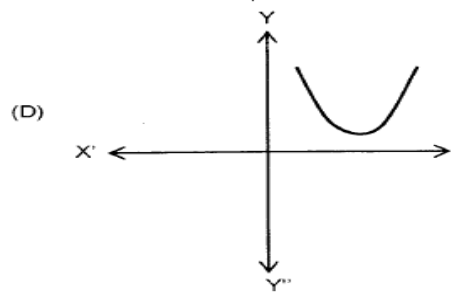
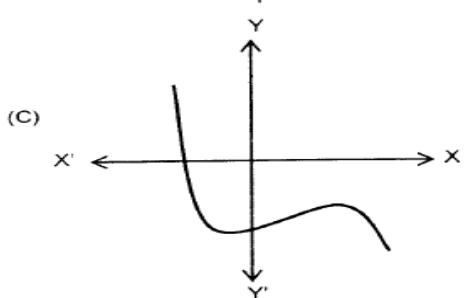
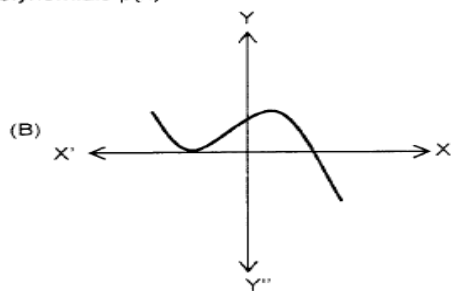
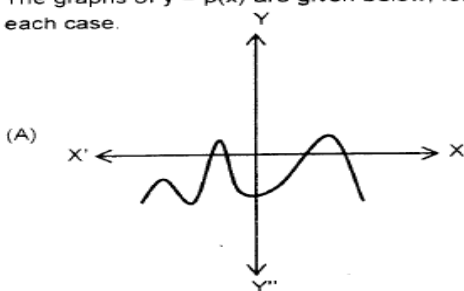
To plot a polynomial on the graph we assign the polynomial in  $x$  to  $y$ . Then we try to find the values of  $y$  by assuming certain values of  $x$ . The value of  $x$  for which the curve cuts the  $x$  axis is the solution of zero of the polynomial.



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## Practice Session II

Q.1 The graphs of  $y = p(x)$  are given below, for some polynomials  $p(x)$ . Find the number of zeroes of  $p(x)$  in each case.



## Home Session

Q1. Write the zeroes of the following polynomial

(i)  $x^2 + 2x + 1$

(ii)  $x^2 - x - 6$

(-1, -1), (-2, 3)

Q2. Find the zeroes of the following quadratic polynomials and verify the relationship with the coefficients.

(i)  $2x^2 - 9 - 3x$

(ii)  $2x^2 - 3x + 5x$

(iii)  $4u^2 + 8u$

(iv)  $3x^2 - x - 4$

(v)  $x^2 - 2x - 8$

(vi)  $t^2 - 15$

(-3/2, 3), (0, -1), (0, -2), (-1, 4/3), (4, -2), (0, 2)

Q3. Is  $x = -2$  a solution of the equation  $x^2 - 2x + 8 = 0$ ?

(No)

Q4. Write the polynomial whose zeroes are -5 and 4.

( $x^2 + x - 20$ )

Q5. Write the polynomial whose zeroes are 1 and -9.

Q6. Write the polynomial whose zeroes are  $2/3$  and  $-1/3$ .

( $9x^2 - 3x - 2$ )

Q7. Write the polynomials. Whose zeros are  $2 + \sqrt{3}$  and  $2 - \sqrt{3}$ .

( $x^2 - 4x + 1$ )

Q8. The graph of  $y = f(x)$  is shown below. Find the number of zeroes of  $f(x)$ .

(2 zeroes)

Q9. Find the polynomial whose zeroes are  $\sqrt{5}$  and  $\sqrt{3}$ .

( $x^2 - (\sqrt{5} + \sqrt{3})x + \sqrt{15}$ )

Q10. Find the zeroes of  $\sqrt{3}x^2 - 8x + 4\sqrt{3}$  and verify the relationship between the zeroes and coefficients.

( $2/\sqrt{3}$ )

Q11. If  $a$  and  $b$  are zeroes of  $x^2 - 3x + 2$ , form an equation whose zeroes are  $a + b$  and  $ab$ .

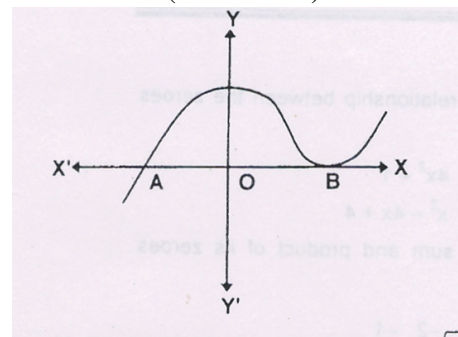
( $3x$ )

Q12. If one of the zeroes of  $(k - 1)x^2 + kx + 1 = 0$  is  $-2$ , find  $k$  and the other zero.

( $k = 3/2, -1$ )

Q13. For what value of  $k$  the zero of  $2x^2 - x + k$  is 4.

( $k = -28$ )



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Q14. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $t(x) = ax^2 + bx + c$ , evaluate:

(i)  $\alpha + \beta$       (ii)  $(\alpha + \beta)^2$       (iii)  $\frac{1}{\alpha} + \frac{1}{\beta}$       (iv)  $\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2$

(v)  $\alpha^3 + \beta^3$       (vi)  $\alpha^4 + \beta^4$

$(-b/a), (b^2/a^2), (-b/c), (b^2 - 4ac / c^2), (bc^3 - ab^2 + 4a^2bc / a^2c^2), [(b^2 - ac)^2 - ac / a^2]$

Q15. If  $\alpha$  and  $\beta$  are the zeros of the polynomial  $p(y) = y^2 + ay + b$ , form a quadratic polynomial whose zeros are  $(\alpha + \beta)^2$  and  $(\alpha - \beta)^2$ . [  $x^2 - (2a^2 - 4b)x + a^2(a^2 - 4b)$  ]

Q16. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $p(x) = 5x^2 - 4x + 1$ , find the value of  $\alpha^2\beta + \alpha\beta^2$  (4/25)

Q17. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $p(z) = 4z^2 - 3z + 4$ , find the value of: (45/16)

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 3\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 2\alpha\beta$$

Q18. If  $\alpha$  and  $\beta$  are the zeros of the quadratic polynomial  $f(x) = 25x^2 - 12x + 20$ , find a quadratic polynomial whose zeros are  $1/(\alpha + \beta)$  and  $(\alpha + \beta)$  [  $300x^2 - 769x + 300$  ]

Q19. If  $\alpha$  and  $\beta$  are zeroes of the quadratic equation  $x^2 + kx + 12$  such that  $\alpha - \beta = 1$  find the value of  $k$ . (+7)

